1.

The hotel that Tom and Tane are staying in is having a promotion at the beach. Each day the hotel staff bury prizes in random locations over an area of 100 m^2, and each hotel guest is allocated a 1 m^2 section of this area to dig in. The number of prizes buried varies each day, with the mean number of prizes buried per day being 60.

Suppose the number of prizes found on the beach each day can be modelled by a Poisson distribution, where the mean number of prizes per 100 m^2 is 60.

Tom and Tane decide to combine their search area so they have a total of 2 m^2 to search.

(a) Calculate the probability that they find no more than 2 prizes in their 2 m^2 area of beach.

(b) Calculate the probability that Tom and Tane find at least one prize on three out of the five days they are staying at that hotel.

2.

One day Tom and Tane go surfing at a local beach. Tom manages to ride a wave, on average, 7 out of every 10 times he tries. Tane manages to ride a wave, on average, 6 out of every 10 times he tries.

Calculate the probability that Tom and Tane each ride a total of 2 of the next 5 waves.

State what probability distribution model you use to solve this problem and justify your choice, including any assumptions you make.

3.

Clare sells her chicken eggs to a supermarket. She sends the eggs in boxes each containing 10 cartons. The probability that any carton contains an egg with a cracked shell is found to be 0.05. If more than two cartons contain cracked shells then the whole box of 10 cartons is rejected. Assume that eggs with cracked shells occur independently.

Calculate the probability that a randomly selected **box** is rejected.

4.

Clare's ducks are free-range, and she has to collect their eggs from all around the farm. On average she finds two duck eggs in a 10 m^2 area.

Assume that a Poisson distribution can model the probability of Clare finding duck eggs.

Calculate the probability that Clare finds at least twelve duck eggs in a randomly selected **30 m^2** area of her farm.

5.

On Clare's farm, a ferret attacks the chickens on average three times every six months.

Calculate the probability that in each of **two consecutive months** a ferret attacks the chickens at least once. Assume that the occurrence of ferret attacks in any given month is independent of the occurrence of ferret attacks in any other month.

6. Double-yolk eggs occur independently at a rate of approximately three per 1 000 eggs. Ten eggs are chosen randomly. Let the random variable X represent the number of eggs in the ten that have double yolks.

Name the probability distribution model for X and give the parameter(s) of this distribution. Justify your answer.

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7. A road service centre finds that on average there are three callouts per hour for assistance with changing flat tyres. The number of callouts received by the service centre for changing flat tyres can be modelled by a Poisson distribution.

Find the probability that there will be fewer than two callouts received by the road service centre for changing flat tyres in any given **20-minute interval**.

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8. Service centre records show that 5% of all callouts are for changing flat tyres. Assume all callouts for changing flat tyres occur independently.

Calculate the probability that more than two of the next ten callouts are for changing flat tyres.

Answers

1.

Poisson: $\lambda = \frac{60}{50} = 1.2$

$P(X \leq 2) = 0.87948$ (GC)

$= 0.8795$ (tables)

Poisson ($\lambda = 1.2$)
$P(X \geq 1) = 0.6988$ (0.69881, GC)

Binomial ($\pi = 0.6988$, $n = 5$)

$P(X = 3) = \binom{5}{3} \times 0.6988^3 \times 0.3012^2$

$= 0.3096$ ($= 0.30956$, GC)

2.

Binomial :

- Probability is constant at 0.7 for each trial for Tom and 0.6 for Tane.
- There are only two outcomes, catch a wave or not.
- There is a fixed number of trials: 5 waves.
- We must assume that Tane catching a wave is independent of him catching the next wave he tries for, and similarly for Tom.

We must assume Tom catching a wave is independent of Tane catching a wave.

Tom $n = 5$, $p = 0.7$,
$P(X = 2) = 0.1323$

Tane $n = 5$, $p = 0.6$,
$P(X = 2) = 0.2304$

P(each catch 2 waves)
$= 0.2304 \times 0.1323$
$= 0.03048192$

Binomial distribution with $p = 0.05$, $n = 10$

$P(X > 2) = 0.0116$

(GC: 0.01151)

3.

Poisson distribution

$\lambda = 6$

$P(X \geq 12) = 0.02$

(GC: 0.02011)

4.

Poisson distribution

$\lambda = 0.5$

$P(X \geq 1) = 0.3935$

P (two consecutive months)

$= 0.3935 \times 0.3935$

$= 0.1548$

5.

Binomial Distribution because:

- fixed number of trials, 10 eggs
- there are only two outcomes for each trial / binary trials, double-yolk or not
- probability of a success on each trial is constant, P(double-yolk) = 0.003
- events / trials are independent, occurrences of double-yolk eggs are independent / given.

Parameters are (n =) 10, (p =) 0.003

OR

$n = 10$, $p = 0.003$

6.

Poisson distribution

$P(x < 2; \lambda = 1)$

$= 0.7358$

(GC: 0.73575)

7.

Binomial distribution

$P(X > 2; n = 10, \pi = 0.05)$

$= 1 - P(X \leq 2)$

$= 1 - 0.98849$

$= 0.0116$

(GC: 0.01151)

8.