## PROBABILITY

Name:


LEADING TO 1.13-PROBABILITY

This booklet is to lead you through the probability unit leading through to the first internal at the beginning of next year. This year, we will look at skills you need and then have a practice at the beginning of the year in preparation for the internal.

The aim of this unit is:
You need to investigate a situation involving elements of chance. ie you're going to look at an experiment and write a report on it.

Vocabulary List - please fill in

| Sample Space |  |
| :--- | :--- |
| Theoretical Probability |  |
| Relative frequency |  |
| Outcome |  |
| Sample size |  |
| Simulation |  |
| Likelihood |  |
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## THEORETICAL PROBABILITY

## Definition of Theoretical Probability

Probability is a likelihood that an event will happen.
We can find the theoretical probability of an event using the following ratio:
$\mathrm{P}($ event $)=\frac{\text { Number of favorable outcomes }}{\text { Total number of possible outcomes }}$
Let's do a couple of examples.

## Example 1

If we toss a fair coin, what is the probability that a tail will show up?
Solution:
Tossing a tail is the favorable outcome here.
When you toss a coin there are only 2 possible outcomes: a Head or a Tail
So the options for tossing a tail are 1 out of 2.
We can also represent probability as a decimal or as a percent.
So, $P($ tail $)=\frac{1}{2}=0.5=50 \%$.

## Example 2

A bag contains 20 marbles. 15 of them are red and 5 of them are blue in color. Find the probability of picking a red marble.
Let's first answer a few questions here:
If I am going to randomly pick a marble from the bag then what results can I have:
I'll either pick a red marble or a blue one.
My next question is what the chances of picking a red marble are:
There are 15 red marbles and just 5 blue marbles.
It's obvious that we have three times as many red marbles as blue marbles.
So, the chance of picking a red marble is more than that of the blue one.
Therefore, the probability of picking a red marble is:
$\mathrm{P}($ red marble $)=\frac{\text { Number of red marbles in the bag }}{\text { Total number of marbles in the bag }}=\frac{15}{20}=\frac{3}{4}=0.75=75 \%$

## Example 3

Find the probability of getting a sum of 7 when you roll two dice.
Two dice are being rolled. The possible outcomes are as follows:
Let's use the representation ( $\mathrm{a}, \mathrm{b}$ ) for the outcomes where $\mathrm{a}=$ number on dice 1 and $\mathrm{b}=$ number on dice 2.
$(1,1),(1,2),(1,3),(1,4),(1,5),(1,6)$, $(2,1),(2,2),(2,3),(2,4),(2,5),(2,6)$,
$(3,1),(3,2),(3,3),(3,4),(3,5),(3,6)$,
$(4,1),(4,2),(4,3),(4,4),(4,5),(4,6)$,
$(5,1),(5,2),(5,3),(5,4),(5,5),(5,6)$,
$(6,1),(6,2),(6,3),(6,4),(6,5),(6,6)$

There are 36 possible outcomes in all.
The question is when you roll two dice, what are the chances of getting a sum of 7 ?
From the list above identify the pairs with outcomes that add up to 7.
Let's highlight them this way:
$(1,1),(1,2),(1,3),(1,4),(1,5),(1,6)$,
$(2,1),(2,2),(2,3),(2,4),(2,5),(2,6)$,
$(3,1),(3,2),(3,3),(3,4),(3,5),(3,6)$,
$(4,1),(4,2),(4,3),(4,4),(4,5),(4,6)$,
$(5,1),(5,2),(5,3),(5,4),(5,5),(5,6)$,
$(6,1),(6,2),(6,3),(6,4),(6,5),(6,6)$
Observe that the pairs along the main diagonal add up to 7. There are 6 such pairs.
So, the probability of getting a sum of 7 when we roll two dice is:
$P($ sum 7$)=\frac{\text { Number of pairs with a sum of } 7}{\text { Total number of possible outcomes }}=\frac{6}{36}=\frac{1}{6} \approx 0.17=16.67 \%$

## RELATIVE FREQUENCY

How often something happens divided by all outcomes.
Example: Your team has won 9 games from a total of 12 games played: the Frequency of winning is 9
the Relative Frequency of winning is $9 / 12=75 \%$
All the Relative Frequencies add up to 1 (except for any rounding error).

## Example: Travel Survey

92 people were asked how they got to work:
35 used a car, 42 took public transport, 8 rode a bicycle, 7 walked
The Relative Frequencies (to 2 decimal places) are:
Car: $35 / 92=0.38$
Public Transport: 42/92 $=0.46$
Bicycle: $8 / 92=0.09$
Walking: 7/92 = 0.08
$0.38+0.46+0.09+0.08=1.01$
(It would be exactly 1 if we had used perfect accuracy),

## SO RELATIVE FREQUENCY IS WHAT DOES HAPPEN

## How can you tell which is experimental and which is theoretical probability?

## Experimental:

You tossed a coin 10 times and recorded a head 3 times, a tail 7 times

$$
\begin{gathered}
P(\text { head })=3 / 10 \\
P(\text { tail })=7 / 10
\end{gathered}
$$

Theoretical:
Toss a coin and getting a head or a tail is $1 / 2$.

$$
\begin{aligned}
& \mathrm{P}(\text { head })=1 / 2 \\
& \mathrm{P}(\text { tail })=1 / 2
\end{aligned}
$$

## Equally likely outcomes

Largest dice number theory
List the largest result for each vertex of the dice (8 total)


Find the probability of ...

1) P (largest of 2 ) $=$
2) $P($ largest of 3$)=$
3) $P($ not a largest of 6$)=$
4) $P$ (largest of more than 4)=

I roll a regular 10 sided die.
8) $P(6)=$
9) $P(2$ or 10$)=$
10) $P(4$ or 5 or 7$)=$
11) $P($ multiple of 3$)=$
12) $P($ an odd $)=$
13) $P(14)=$
14) $P($ not a 7$)=$
15) $P($ A factor of 10$)=$

## Probability scale

Probability is a measure of the chance of an event happening.
It can be a decimal, fraction or percentage (decimal or fraction are best)
Probabilities MUST be between 0 and 1 (inclusive)


Experimental Probability = Relative frequency


What is the probability of shooting
an arrow into each of the target zones?
How can we find this?


## Long run relative frequencies

If a probability is found by experiment it is an estimate.
To improve the accuracy of the estimate we repeat the number or trials
There are 12 balls in the box
How many of each colour are there?

| Number | of trials <br>  <br>  <br>  |
| :--- | :--- |


| Outcome | Frequency | Relative <br> frequency | Expected <br> number |
| :--- | :--- | :--- | :--- |
| Red | 5 | 0.5 | 6 |
| Blue | 3 | 0.3 | 3.6 |
| Green | 2 | 0.2 | 2.4 |
| Total | 10 | 1 | 12 |




「he experiment is repeated 100 times...

There are 12 balls in the box
How many of each colour are there?

| Number |
| :---: |
|  |
|  |
|  |
| $=$ |


| Outcome | Frequency | Relative <br> frequency | Expected <br> number |
| :--- | :--- | :--- | :--- |
| Red | 36 | 0.36 | 4.32 |
| Blue | 29 | 0.29 | 3.48 |
| Green | 35 | 0.35 | 4.2 |
| Total | 100 | 1 | 12 |


| Draw <br> number | Result |
| :--- | :--- |
| 1 | Red |
| 2 | Green |
| 3 | Red |
| 4 | Green |
| 5 | Red |
| 6 | Red |
| 7 | Green |
| 8 | Green |
| 9 | Green |
| 10 | Blue |




The experiment is repeated 3000 times...
There are 12 balls in the box
How many of each colour are there?

| Number | of trials |
| :---: | :---: |
|  | $=3000$ |


| Draw <br> number | Result |
| :--- | :--- |
| 1 | Green |
| 2 | Blue |
| 3 | Blue |
| 4 | Blue |
| 5 | Green |
| 6 | Green |
| 7 | Blue |
| 8 | Green |
| 9 | Blue |
| 10 | Red |


| Outcome | Frequency | Relative <br> frequency | Expected <br> number |
| :--- | :--- | :--- | :--- |
| Red | 997 | 0.33233333 | 3.988 |
| Blue | 972 | 0.324 | 3.888 |
| Green | 1031 | 0.34366667 | 4.124 |
| Total | 3000 | 1 | 12 |



As number of repeats increases, the accuracy also increases

## Long run relative frequencies

| Flip | Result | H's so far | P(Head) |
| :--- | :--- | :--- | :--- |
| 1 | T | 0 | 0.00 |
| 2 | H | 1 | 0.50 |
| 3 | T | 1 | 0.33 |
| 4 | T | 1 | 0.25 |
| 5 | H | 2 | 0.40 |
| 6 | T | 2 | 0.33 |
| 7 | T | 2 | 0.29 |
| 8 | T | 2 | 0.25 |
| 9 | H | 3 | 0.33 |
| 10 | T | 3 | 0.30 |
| 11 | H | 4 | 0.36 |
| 12 | H | 5 | 0.42 |
| 13 | T | 5 | 0.38 |
| 14 | H | 6 | 0.43 |
| 15 | T | 6 | 0.40 |
| 16 | H | 7 | 0.44 |



## Long run relative frequencies

Roll a die at least 20 times, calculating the long run relative frequency for 5 or 6

| Roll | Result | 5,6 's so <br> far | $P(5,6)$ |
| :--- | :--- | :--- | :--- |
| 1 |  |  |  |
| 2 |  |  |  |
| 3 |  |  |  |
| 4 |  |  |  |
| 5 |  |  |  |
| 6 |  |  |  |
| 7 |  |  |  |
| 8 |  |  |  |
| 9 |  |  |  |
| 10 |  |  |  |
| 11 |  |  |  |
| 12 |  |  |  |
| 13 |  |  |  |
| 14 |  |  |  |
| 15 |  |  |  |



To see how this done on excel have look at :
https://www.youtube.com/watch?v=B3i3W0gPKNg
$\vec{n}$

## USING THE PPDAC CYCLE

## Are You a Data Detective?


$=\Rightarrow \quad$ - $\quad$ - ała detectives use PPDAC
Problem
What question do you want answered, or what do you want to find out about?

## Plan

Decide what information is needed and how to collect, record and analyse the data.

## Data

Collect the data. If the data needs to be collected through a sample or questionnaire, think about the questions you are going to ask and how you are going to analyse the data.

Analysis
Organise the raw data into tables and construct appropriate graphs.

## Conclusion

Describe what the graphs show and answer the question.

## PROBLEM

Setting up a question to ask

## Achieved:

posing a question to explore a situation and set up a simulation involving elements of chance

## Merit:

With justification - why did you choose that question?

## Excellence:

With insight - as above and where could it lead? Why is it relevant?

## PLAN

Make a comprehensive plan of what you are going to do

## Achieved:

Plan an experiment to explore the situation
Describe the set of all possible outcomes (sample space)
Choose an appropriate sample size

## Merit:

As Achieved, with justification
Why did you choose the sample size?
Description of why for all parts of your plan

## Excellence:

Indicates in the procedure how the data analysis will answer the question Consider the effect of related variables - what else could happen?

## DATA

Collect your data by doing the simulation
Same for all levels

## ANALYSIS

Drawing graphs
Calculating statistics

## Achieved:

Selecting and using appropriate displays including experimental probability distributions
Identifying and communicating patterns in the data
Comparing discrete theoretical distributions and experimental distributions as appropriate

## Merit:

All the above using evidence linking it all to the situation

## Excellence:

Use insight showing an understanding of applications of probability Creates appropriate displays, one of which is an experimental probability distribution, showing different features of the data, including all outcomes, individual values and summary features

## CONCLUSION

A summary

## Achieved:

Identifies at least two features of the data in context;
Answers the investigative question with evidence from the data;

## Merit:

Use evidence from the data in context to answer the question
Answers the investigative question with evidence from the probability distribution; Justifies the conclusion with references to features of the data displays.
Reflection on the prediction could be part of the justification;

## Excellence

Demonstrates an understanding of applications of probability, explaining the most important features of the data and how they relate to the context;
Answers the investigative question with evidence from the probability distribution and justifies the conclusion with references to visual features of the data displays. An in-depth reflection of the prediction could be part of the evidence;
Reflects on the investigation, including possible effects of other factors, and/or makes additional conjectures about future performance.

## KEY TIPS

Be familiar with outcomes based on using dice, coins, and playing cards.
Practise using simulations to solve probability problems.
Record your data carefully.
Check you have addressed every requirement that was asked for in planning your investigation.

Use appropriate graphs to display your data. They should enable you to show features in relation to your investigation.

Summarise your findings, referring to the steps you took to come to your conclusion.
You may be expected to complete probability trees and use these.
Multiply the probabilities along the branches of tree diagrams if you are asked for the probability that an event AND another event occur.

Add the probabilities at the end of the branches if you are asked for the probability that an event OR another event occurs.

Probabilities are best written as fractions, but may be written as a decimal or percentage. If written as a percentage, you must include the percentage sign.

Probability must not be written as a ratio.

## Writing Frame

## Problem

Set a question to research
I am going to investigate
I know I will get
I think I will probably get
Plan
I am going to ......
Data
Table of results
Chart / display of results

## Analysis

Looking at my . . . . . graph I can see

## Conclusion

Discuss findings and answer your research question
You should be using TTRC to solve these problems and merge it into PPDAC

- Tool -
definition of the probability tool
how the tool models the situation
- Using calc random number function.
- Using the first 2 digits to represent each traffic light.
- If the digit is 1 or 2 or 3 the light is green, otherwise it is red.
- Trial -

Definition of a trial
definition of a successful outcome

- Generate one random number to represent one trip to school.
- Record the result.
- Example 0.386 will be green (3) red (8) ignore the 6.
- Results

Statement of how the results will be tabulated giving an example of a successful outcome and an unsuccessful outcome
Statement of how many trials will be carries out

- Set results out in a table
- Calculations

Statement of how the calculation is done for the conclusion

## ASSESSMENT SCHEDULE

## Evidence/Judgements for Achievement

- Poses an appropriate investigative question;
- Plans a suitable experiment;
- Describes the set of possible outcomes and chooses a sample size;
- Gathers data as per plan;
- Creates at least two appropriate data displays, one of which is an experimental probability distribution;
- Identifies at least two features of the data in context;
Answers the investigative question with evidence from the data;


## Evidence/Judgements for

 Achievement with Merit- Poses an appropriate investigative question and plans a suitable experiment;
- Describes the set of possible outcomes, and sample size, with reasons why specific plan elements were chosen;
- Gathers data as per plan;
- Creates at least two appropriate data displays, one of which is an experimental probability distribution;
- Compares experimental probability with theoretical probability.
- Identifies at least two features of the data in context;
- Answers the investigative question with evidence from the probability distribution;

Justifies the conclusion with references to features of the data displays.

- Reflection on the prediction could be part of the justification;

Evidence/Judgements for Achievement with Excellence

- Poses an insightful investigative question and plans a comprehensive experiment;
- Describes the set of possible outcomes, and sample size, with statistical reasons why specific plan elements were chosen, for example, indicates in the procedure how the data analysis will answer the question, and/or considers the effect of related variables;
- Gathers data as per plan;
- Creates appropriate displays, one of which is an experimental probability distribution, showing different features of the data, including all outcomes, individual values and summary features;
- Calculates the theoretical probabilities and compares to the experimental outcome.
- Demonstrates an understanding of applications of probability, explaining the most important features of the data and how they relate to the context;
- Answers the investigative question with evidence from the probability distribution and justifies the conclusion with references to visual features of the data displays. An indepth reflection of the prediction could be part of the evidence;
- Reflects on the investigation, including possible effects of other factors, and/or makes additional conjectures about future performance.


## Activity: Quiz or no quiz

Introduction: This activity involves students exploring the outcomes of tossing a coin repeatedly to decide whether there is a quiz or not.

## Problem

## Plan

## Data

The teacher will make a deal with the students:
Start with a counter at the top. Toss a coin four times. Heads means move the counter down and left. Tails means move the counter down and right.

- Cups 1, 2, or 5 = No Quiz
- Cups 3 or 4 = Quiz

Is this a fair deal?
Students predict how many times the counter will end up in each cup if the experiment is done 100 times.


- Toss the coin 4 times and follow the path to see which cup the counter ends up in.

Do the experiment 16 times and record the number of times the counter ends up in cup1, cup2, cup3, cup4 and cup5, therefore find the experimentally estimated probabilities that it ends up in cup1, cup2, cup3, cup4, and cup5.

Combine the results with other students and find the experimentally estimated probabilities.

- Check: Is this a fair deal? How close was their prediction?


## Analysis

Lead students to calculate the theoretical probabilities to verify their experimentally estimated results

Calculate the theoretical probability that it ends up in cups 1, 2, or 5 ("OR/PLUS" concept or by counting all the outcomes )
Calculate the theoretical probability that it ends up in cups 3 or 4 ("OR/PLUS" concept)

Systematically list all the possible outcomes (the sample space) for landing in each cup.

List the outcomes of the four tosses that end up in cup1: HHHH ----4 heads

- Find the number of ways to get to this cup1 (4 heads)---- 1 way
- List the outcomes of the four tosses that end up in cup2: HHHT, HHTH, HTHH etc ----3 heads 1 tail
- Find the number of ways to get to this cup2 (3 heads and 1 tail)-----4 ways
- List the outcomes of the four tosses that end up in cup3----2 heads 2 tails
- Find the number of ways to get to this cup3 ( 2 heads and 2 tails)----- 6 ways
- List the outcomes of the four tosses that end up in cup4----1 head 3 tails
- Find the number of ways to get to this cup4 (1 head and 3 tails)------4 ways
- List the outcomes of the four tosses that end up in cup5----4 tails
- Find the number of ways to get to this cup5 (4 tails)-----1 way
- Introduce Pascal Triangle to find the number of ways to get to a particular combination
- Use the concept of "AND/Independence/Multiplication" to calculate
(a) the theoretical probability of getting 4 heads---1/16
(b) the theoretical probability of getting 3 heads and 1 tail -----4/16
(c) the theoretical probability of getting 2 heads and 2 tails----6/16
(d) the theoretical probability of getting 1 head and 3 tails -----4/16
(e) the theoretical probability of getting 4 tails ------1/16

| $x$ | 1 | 2 | 3 | 4 | 5 | Total |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathrm{P}(x)$ | $1 / 16$ | $4 / 16$ | $6 / 16$ | $4 / 16$ | $1 / 16$ | 1 |

This is also an informal introduction to binomial distribution

- Calculate the theoretical probabilities of getting into cups 1, 2, or 5
- Calculate the theoretical probabilities of getting into cups 3 or 4
- Compare the experimentally estimated probabilities with the theoretical probabilities, the experimentally estimated probabilities should approach the theoretical probabilities as the sample size increases.

Students construct a version of the game they think is fair.

## ACTIVITY - BOY OR GIRL?

Based on current birth records in New Zealand there are 104 boys born for every 100 girls. The probability that a child is a boy is therefore approximately 0.51 and a girl 0.49 .

This means that in a family of three children the probability that all three children are boys is 0.13 the probability that exactly one is a girl is 0.38 the probability that at least one is a girl is 0.87 .

The mathematics of theoretical and experimental probability is used in many other areas of medical science where the outcome of many procedures and diseases is a matter of chance.

Exercise:
(a) Using a probability tree, confirm the theoretical probabilities stated above are correct.
(b) Use a computer simulation written in Excel. How close are your results?

## HCTINTY - CNME OF 'PRISOMERS'

## Game resources

- Paper and pens
- Two different coloured 6 -sided dice between two students. The game is played in pairs. Alternatively, you can play as a whole class.

Grid for game play (each student draws up before they start playing)

| 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
|  |  |  |  |  |  |  |  |  |  |  |

## Game description

Each student draws 10 dashes in the space below their 'totals strips' - more than one dash may be placed under any of the numbers. Players take turns to throw the pair of dice and note the total. If the total on either players grid has dashes marked under it, they can cross out ONE dash from under that total. The first person to cross out all the 10 dashes off their grid is the winner.

## Analysis

The students play the game in pairs and write some questions that they might want to ask about this game. As they play the game ask questions that focus on the probabilities of the game:

- What do the popular totals seem to be?
- Where on the board are the totals that don't seem to occur very often?
- Why might some totals occur more often than others?
- If you were to play the game again, where might you place your dashes?

Play a second game trying to place their dashes in the 'best' places. After the students have played the game ask them to comment on the effectiveness of their placements.

| 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
|  |  |  |  |  |  |  |  |  |  |  |

Pose the question: 'How can we determine the likelihood of different totals?' Discuss the need to have a 'reasonable' number of trials and the ways that you could keep track of the totals.

We need a systematic way to represent all possible outcomes. The following grid will probably help:

| + | $\mathbf{1}$ | $\mathbf{2}$ | $\mathbf{3}$ | $\mathbf{4}$ | $\mathbf{5}$ | $\mathbf{6}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathbf{1}$ | 2 | 3 | 4 | 5 | 6 | 7 |
| $\mathbf{2}$ | 3 | 4 | 5 | 6 | 7 | 8 |
| 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| $\mathbf{5}$ | 6 | 7 | 8 | 9 | 10 | 11 |
| 6 | 7 | 8 | 9 | 10 | 11 | 12 |

Statement: 'Of the 36 possible outcomes, 10 of them should give totals less that 6.'

Is this correct?

Students should now be asked to play another game, hopefully with a better winning strategy. Does their new strategy help? Students should consider variation between the experimental results and conclusions reached using the table above.

| 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
|  |  |  |  |  |  |  |  |  |  |  |


| 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  |  |  |  |  |  |  |  |


| 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
|  |  |  |  |  |  |  |  |  |  |  |

## Follow up

You may wish to use a computer applet or a spreadsheet to show the distribution of totals for larger samples of the sum of two 6 -sided dice.

## HCTIITY - CMES OF "CREEDY PIG"

## Game resources

Paper and pens
One 6 -sided die between two students.
The game is played in pairs.
Alternatively, you can play as a whole class.
Grid for game play (shown beside)
Game description
There are several versions of the game of 'Greedy Pig'. The version we will be playing is that the winner of the game is the person who has the highest total after 10 rounds. Each round consists of rolling the die until a ' 2 ' shows up. At the start of each round all students are standing. If a student is standing when a ' 2 ' is rolled they score zero for that round, otherwise they score a round total that is the sum of the rolls of the die until they decide to sit down. At the end of each round students add their round total to their running total.

## Game Play

Each student must decide on a strategy that they must stick to during the whole game.

For example:
sit down after 3 rolls, if you are still 'alive' sit down when you first get a total of 20, if you are still 'alive'
sit down after a ' 6 ' is rolled, if you are still alive.
Play the game several times, using the same strategy for at least two games.

What is the 'best' (winning) strategy ????

Strategy: $\qquad$

| Round | Round <br> total | Running <br> total |
| :---: | :---: | :---: |
| 1 |  |  |
| 2 |  |  |
| 3 |  |  |
| 4 |  |  |
| 5 |  |  |
| 6 |  |  |
| 7 |  |  |
| 8 |  |  |
| 9 |  |  |
| 10 |  |  |
| Total |  |  |

## Strategy:

$\qquad$

| Round | Round <br> total | Running <br> total |
| :---: | :---: | :---: |
| 1 |  |  |
| 2 |  |  |
| 3 |  |  |
| 4 |  |  |
| 5 |  |  |
| 6 |  |  |
| 7 |  |  |
| 8 |  |  |
| 9 |  |  |
| 10 |  |  |
| Total |  |  |

Strategy: $\qquad$

| Round | Round <br> total | Running <br> total |
| :---: | :---: | :---: |
| 1 |  |  |
| 2 |  |  |
| 3 |  |  |
| 4 |  |  |
| 5 |  |  |
| 6 |  |  |
| 7 |  |  |
| 8 |  |  |
| 9 |  |  |
| 10 |  |  |
| Total |  |  |

Strategy: $\qquad$

| Round | Round <br> total | Running <br> total |
| :---: | :---: | :---: |
| 1 |  |  |
| 2 |  |  |
| 3 |  |  |
| 4 |  |  |
| 5 |  |  |
| 6 |  |  |
| 7 |  |  |
| 8 |  |  |
| 9 |  |  |
| 10 |  |  |
| Total |  |  |

Strategy: $\qquad$

| Round | Round <br> total | Running <br> total |
| :---: | :---: | :---: |
| 1 |  |  |
| 2 |  |  |
| 3 |  |  |
| 4 |  |  |
| 5 |  |  |
| 6 |  |  |
| 7 |  |  |
| 8 |  |  |
| 9 |  |  |
| 10 |  |  |
| Total |  |  |

Strategy: $\qquad$

| Round | Round <br> total | Running <br> total |
| :---: | :---: | :---: |
| 1 |  |  |
| 2 |  |  |
| 3 |  |  |
| 4 |  |  |
| 5 |  |  |
| 6 |  |  |
| 7 |  |  |
| 8 |  |  |
| 9 |  |  |
| 10 |  |  |
| Total |  |  |

## LCTIITY - CHME OF "PIPER, SCISSORS, ROCK"

## Game resources

- Paper and pens


## Game description

The game paper scissors rock is often played by children. It is when at the same time two people use their hands to display one of three hand signals. The first is paper (hand flat), rock (hand is in a fist) or scissors (hand makes scissors). The rules are:

- Paper beats rock
- Scissors beats paper
- Rock beats scissors.

Problem - 'Miriam says that she always does the rock, because this makes it more likely to win."

Set up and carry out an experiment to decide if Miriam is right or not. Record your results below.

Another game of paper, scissors, rock

Stephen plays paper scissors rock with his brother to choose who will do the dishes. He decides to do a 'best of three'. If a person wins two or more times then the other person does the dishes. If it is a draw (no player has majority wins) then their Dad does the dishes.

Problem - 'Who is more likely to do the dishes - the brothers or their Dad?'

Set up and carry out an experiment to answer the question. Record your results below.

## ACTIVITY - GIME OF 'MMKE THE HIGHEST TOTAL'

Game resources
A 6-sided die, paper and pens

Grid for game play (shown beside)

Game description
Karen is playing a game.
The winner is the person with the highest total at the end of ten rounds.

Each round, a die is rolled three times. After each roll, each player decides where they want to place the number showing on the die - the hundreds, the tens or the ones

| Round | 100 s | 10 s | 1 s |
| :---: | :---: | :---: | :---: |
| 1 |  |  |  |
| 2 |  |  |  |
| 3 |  |  |  |
| 4 |  |  |  |
| 5 |  |  |  |
| 6 |  |  |  |
| 7 |  |  |  |
| 8 |  |  |  |
| 9 |  |  |  |
| 10 |  |  |  |
| Total |  |  |  | column position.

Remember, the aim is to get the highest total in 10 rounds played.

Analysis
What is the 'best' (winning) strategy ????

Strategy chosen:

| Round | 100s | 10 s | 1 s |
| :---: | :---: | :---: | :---: |
| 1 |  |  |  |
| 2 |  |  |  |
| 3 |  |  |  |
| 4 |  |  |  |
| 5 |  |  |  |
| 6 |  |  |  |
| 7 |  |  |  |
| 8 |  |  |  |
| 9 |  |  |  |
| 10 |  |  |  |
| Total |  |  |  |


| Round | 100 s | 10 s | 1 s |
| :---: | :---: | :---: | :---: |
| 1 |  |  |  |
| 2 |  |  |  |
| 3 |  |  |  |
| 4 |  |  |  |
| 5 |  |  |  |
| 6 |  |  |  |
| 7 |  |  |  |
| 8 |  |  |  |
| 9 |  |  |  |
| 10 |  |  |  |
| Total |  |  |  |


| Round | 100 s | 10 s | 1 s |
| :---: | :---: | :---: | :---: |
| 1 |  |  |  |
| 2 |  |  |  |
| 3 |  |  |  |
| 4 |  |  |  |
| 5 |  |  |  |
| 6 |  |  |  |
| 7 |  |  |  |
| 8 |  |  |  |
| 9 |  |  |  |
| 10 |  |  |  |
| Total |  |  |  |


| Round | 100 s | 10 s | 1 s |
| :---: | :---: | :---: | :---: |
| 1 |  |  |  |
| 2 |  |  |  |
| 3 |  |  |  |
| 4 |  |  |  |
| 5 |  |  |  |
| 6 |  |  |  |
| 7 |  |  |  |
| 8 |  |  |  |
| 9 |  |  |  |
| 10 |  |  |  |
| Total |  |  |  |


| Round | 100 s | 10 s | 1 s |
| :---: | :---: | :---: | :---: |
| 1 |  |  |  |
| 2 |  |  |  |
| 3 |  |  |  |
| 4 |  |  |  |
| 5 |  |  |  |
| 6 |  |  |  |
| 7 |  |  |  |
| 8 |  |  |  |
| 9 |  |  |  |
| 10 |  |  |  |
| Total |  |  |  |

## ACTIVITY - 'MLIRPHY's LAW'

## Purpose:

In this unit, students will explore the commonly held believe that if anything bad can possibly happen it will and at the most inopportune time. Students are encouraged to look at events involving chance and predict the likelihood of certain outcomes by both trialling the event and analysing it theoretically.

## Specific Learning Outcomes:

Use simulations to investigate probability in common situations predict the likelihood of outcomes on the basis of an experiment determine the theoretical probability of an event
Description of mathematics:
This unit takes an interesting approach to probability by delving into some commonly held myths that can be thought of as going under the name of Murphy's Law. The basic tenet here is that what can go wrong will go wrong. A series of practical situations are explored experimentally to test a number of Murphy's Laws. Students get a chance to test and explore widely held views under strict, controlled conditions. These experiments are simulations of the real events.
Simulating actual events is an important study in itself. Often we cannot run the actual experiment because it would take too long or would be too expensive. So finding a means to simulate the event provides a cheap but accurate way to determine the probability of the event. One of the advantages of computers is that they can simulate quite complicated situations and carry out trials reasonably quickly.

## Required Resource Materials:

## Car keys

Box
Red, green and orange counters
Dice
Ice-cream containers
Blank cubes
Blank cards
History of Murphy's Law

## Murphy's Law's to investigate:

- The first 'Law' we will investigate s that of Murphy's Law relating to keys. There you are carrying a heavy box of things to the door or to the car boot. You put the box in one arm to hold it while you reach inside your pocket for the keys and, you guessed it, the keys are in the other pocket! So you shift the load onto the other arm to get the keys out or you become a contortionist by trying to get it with the opposite arm. So Murphy's Law for keys says that keys are always in the pocket that you can't reach.
- Murphy's Law of Traffic Lights

If you are in a rush the traffic lights are always red when you get to them.
This can be simulated by putting two red counters, two green counters, and one orange counter into an ice-cream container and then drawing a counter out at random. This gives you the colour of the lights on your arrival (probability of red is $2 / 5=40 \%$ ). Your degree of haste can easily be simulated by rolling a dice (even numbers for rush, odd for lots of time), making up equal numbers of cards with rush and time on them and drawing one from a bag each time, or flipping a coin (heads for rush, tails for lots of time).
In this case students should observe that your degree of haste makes no difference to the likelihood of getting a red light. This brings up the issue of people believing in Murphy's Law because unfavourable events in moments of crisis are much more memorable than favourable events in moments of tranquillity.

- Murphy's Law of Buttered Toast

The more expensive the carpet the greater the chance that the piece of toast that falls off your plate will land butter side down.
This can be simulated easily using a coin for the piece of toast (heads for butter side, tails for not buttered) and a money dice (faces of 10c, 20c, 50c, \$1, 2 \& on the blank face students can roll again). Real bread can be used but you need to be culturally sensitive about the use of food in this context. Note that the outcomes of trials could be organised in a chart, like this:

| Expense of Carpet |  |  |  |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- |
| Landed | $10 c$ | $20 c$ | $50 c$ | $\$ 1$ | $\$ 2$ |
| Butter | 5 | 2 | 5 | 3 | 7 |
| Not Butter | 7 | 5 | 3 | 5 | 5 |

Students should realise that in this case the expense of the carpet has no impact on the theoretical probability of the bread landing butter side down. It is people's recall of unfortunate occurrences that is responsible for this widely accepted corollary of Murphy's Law.

- Murphy's Law of Drawing Pins

If a drawing pin drops on the floor the chance of it landing sharp end up increases as its distance to the nearest bare foot decreases.
Encourage the children to discuss this and then devise an experiment to test it.

- Murphy's Law of Supermarket Queues

Whatever queue you join, no matter how short it looks, will always take the longest for you to get served.
This example is more complex in its organisation. It may be easier to have students act it out.
You may wish to discuss with the students what things might affect how long it takes a supermarket queue to get served. They should have recollections of frustrated parents as the person in front took five minutes to write a cheque, or requested an obscure article that took a supermarket employee ages to find, or the
checkout operator was in training. Write down the factors that affect the time that a person takes to get through a checkout.

- A simple example of Murphy's Law operating in sport is some Captains' total inability to win the coin toss.
At home this week you are to try as hard as you can to emulate the feat of a certain New Zealand cricket captain by losing the toss four times in a row. How easy is this to do?
Experiment to find out.
Write down the results of your experiment and write a statement about how easy it is to lose the toss four times in a row.

